

Braaten-Pisarski Action, Disoriented Chiral Condensate, and Chiral Symmetry Non-Restoration ¹

Ngee-Pong Chang (npccc@cunyvm.cuny.edu)

Department of Physics

City College & The Graduate School of City University of New York
New York, N.Y. 10031

Aug 23, 1993

Abstract

The QCD effective action at high T shows a manifest global chiral symmetry. And calculations show that the order parameter $\langle \bar{\psi}\psi \rangle$ vanishes above T_c .

Is this not evidence for chiral symmetry restoration?

It has been popular to refer to this T_c as chiral symmetry restoration temperature because it fits into our prejudice that chiral symmetry is like an ‘ordered’ state, and at high T it must become disordered. In fact, NJL[1] ground state is not an ordered spin state. I give an example of a generalized chiral broken NJL ground state for which $\langle \bar{\psi}\psi \rangle$ nevertheless vanishes.

The recent scenario of a generic class of disoriented chiral condensate[2] offers another example where $\langle \bar{\psi}\psi \rangle$ in each little domain is nonzero, but the average over all space of $\langle \bar{\psi}\psi \rangle$ vanishes. Such a dcc ground state continues to break chiral invariance.

But how do you reconcile this with the apparent chiral symmetry at high T ?

The Braaten-Pisarski action[3] is a good laboratory to investigate the subtleties of high temperature chiral symmetry. By carrying out a canonical quantization of this highly nonlocal action, I demonstrate how the thermal vacuum at high T conserves the new β -chirality but breaks the old $T = 0$ chirality.

Lattice calculations show that the pion develops a screening[4] mass at high T . Our continuum field theory calculations[5] show that the QCD pion remains massless for all T . I conclude the talk by showing how the hot pion manages to accomodate the two results by propagating in the early universe with a halo.

¹Contributed talk at the 3rd Thermal Fields Workshop held Aug 16 - 27, 1993 at Banff, Canada.
Parts of this work have been supported in part by a grant from NSF and from PSC-BHE of CUNY.

1 Introduction

In these idyllic surroundings and cool environment of Banff, I am pleased to be discussing with you today a different view of the phase transition that takes place at a much hotter temperature, T_c . I am referring to the vanishing of $\langle \bar{\psi} \psi \rangle$ for T above T_c . As mentioned in the abstract, I will give in this talk the background to my contention why in spite of appearances, *the chiral symmetry that we know at $T = 0$ is not restored at high T .*

And yet the QCD effective action at high T is manifestly chiral invariant. Is chiral symmetry not restored? The Braaten-Pisarski effective action[3] is a good laboratory in which to point out the subtleties involved. It is globally chiral invariant, so that it would appear to be consistent with the popular notion of chiral restoration. And yet the quark propagates through a hot medium[6] as if it has a pseudo-Lorentz invariant mass[7], $T' (\equiv g_r \sqrt{C_F} T/2)$. While chiral symmetry at $T = 0$ requires that the fermion be massless, this new chiral symmetry at high T allows for what has been referred to as thermal mass. This thermal mass is there for both QED and QCD.

The Noether charge associated with the high temperature chiral phase is demonstrably different than the usual Noether charge. I have performed a canonical quantization of the BP effective action for the quark field, and will present to you the canonical expansion for the two Noether charges, so that you can judge for yourself.

The implications of a continued breaking of chiral symmetry at high T are, of course, quite astounding. If you believe in a fundamental Higgs field for the Standard Model, then above the (very high) electroweak transition temperature, the vev for the Higgs field vanishes, and the quarks no longer acquire a mass through the Yukawa coupling. The Nambu-Goldstone theorem now forces the pion *to be strictly massless in the early universe* even in the presence of electroweak interactions.

In the concluding section of my talk, I present a picture of the propagation of the pion through the early hot medium, and show how it propagates with light velocity, but acquires a halo.

Presumably, the presence of the $q\bar{q}$ bound state in the early universe will have some new and subtle effect. But as to what that will be, I can only hope someone in the audience will be expert enough to advise me.

2 NJL ground state at $T = 0$

At zero temperature, the pioneering work of Nambu and Jona-Lasinio[1] has taught us how massless fermions manage nevertheless to acquire dynamical mass. The NJL ground state is made up of massless quark-antiquark pairs with the same helicity

$$|vac\rangle = \prod_{p,s} \left(\cos \theta_p - s \sin \theta_p a_{p,s}^\dagger b_{-p,s}^\dagger \right) |0\rangle \quad (1)$$

where s is defined to be ± 1 respectively for R and L helicities. The observed massive quarks are the quasi-particle excitations off this ground state.

$$A_{p,s} = \cos \theta_p a_{p,s} + s \sin \theta_p b_{-p,s}^\dagger \quad (2)$$

$$B_{-p,s} = \cos \theta_p b_{-p,s} - s \sin \theta_p a_{p,s}^\dagger \quad (3)$$

The mass gap associated with these quasi-particles are directly related to the amount of $q\bar{q}$ mixing in the NJL vacuum ($\tan 2\theta_p = m/p$) and may be determined self-consistently from the dynamics by solving the famous gap equation.

The original scale invariant Lagrangian is formally invariant under the global chiral transformation

$$\psi(\vec{x}, t) \rightarrow e^{i\alpha\gamma_5} \psi(\vec{x}, t) \quad (4)$$

The Noether charge Q_5 that generates the phase changes for the massless quark and antiquark operators is given by

$$Q_5 = \frac{1}{2} \int d^3x \psi^\dagger \gamma_5 \psi = -\frac{1}{2} \sum_{p,s} s \left(a_{p,s}^\dagger a_{p,s} + b_{-p,s}^\dagger b_{-p,s} \right) \quad (5)$$

and you can see how the NJL ground state, eq.(1), is not annihilated by this Q_5 .

A signature for this spontaneous breakdown is the nonvanishing of the order parameter, $\langle \bar{\psi} \psi \rangle$. QCD sum rules, as well as lattice calculations and continuum field theory have all demonstrated this. Our earlier calculation[8] showed the connection between dynamical symmetry breaking and bifurcation theory, and led to a universal prediction

$$\langle \bar{\psi} \psi \rangle = -0.0398 N_c N_f \Lambda_c^3 \quad (6)$$

Associated with this breakdown is the presence of the Nambu-Goldstone pion in $T = 0$ QCD. If we could ignore electroweak interactions, then the pion is to be massless. Because of the fermion Yukawa couplings, the electroweak breaking feeds a tree level mass to the quarks, and the QCD pion is no longer massless, and acquires the observed 135 MeV.

As T increases, it has been observed that $\langle \bar{\psi} \psi \rangle$ vanishes at some T_c , and stays zero for T above it. Our calculations[7] show this T_c to be $\Lambda_c e^{2/3}$. In the popular folklore this phase transition is interpreted as chiral symmetry restoration at high temperatures, in line with what happens, say, with the Heisenberg ferromagnet.

It does not have to be so. I quote here a very simple ‘counterexample’ of a generalized NJL ground state that would have $\langle \bar{\psi} \psi \rangle = 0$, and yet is manifestly not chiral invariant. Namely, put a phase factor i in the NJL ground state

$$|vac\rangle' = \prod_{p,s} \left(\cos \theta_p - i s \sin \theta_p a_{p,s}^\dagger b_{-p,s}^\dagger \right) |0\rangle \quad (7)$$

It is an instructive but very simple exercise to check that $\langle \bar{\psi} \psi \rangle$ vanishes with respect to this new ground state. I mean this example to show that $\langle \bar{\psi} \psi \rangle$ is an incomplete order parameter for chiral symmetry breaking. It measures the ‘real’ part of the NJL ground state, and misses out on the ‘imaginary’ part of a generalized NJL state.²

3 Disoriented Chiral Condensate

To be sure, the vanishing $\langle \bar{\psi} \psi \rangle$ at T_c signals a phase transition. If it is not chiral symmetry restoration, then what could it be? I would like to suggest to you that it is a transition to the new

² The full extent of the chirality algebra and the larger set of order parameters associated with the algebra will be described in a forthcoming separate paper.

β -chiral phase, and the generic class of disoriented chiral condensate[2] $(dcc)^3$ is a good realization of this new equilibrium phase.

In this scenario, the ground state (i.e. the universe) above T_c breaks up into many little domains, inside each of which $\langle \bar{\psi} \psi \rangle$ takes a different value, so that when averaged over all domains, $\langle \bar{\psi} \psi \rangle$ vanishes. Under a global chiral transformation, each domain would further tilt, so that in such a scenario, the class of disoriented chiral condensate vacua is not invariant under the α -chiral transformation of eq.(4).

4 Braaten-Pisarski Action

Because I am focussing on the chiral symmetry aspects, I will proceed forthwith to consider only the two fermion sector and set the background gluon field to zero. The form of the BP action⁴ that we shall study then has the form

$$I_{BP} = \int d^4x \left\{ -\bar{\psi}_\beta \gamma_\mu \partial^\mu \psi_\beta - \frac{T'^2}{4} \int dt' \langle \bar{\psi}_\beta(\vec{x}, t) (\gamma_o - \vec{\gamma} \cdot \hat{n}) \psi_\beta(\vec{x} - \hat{n}(t - t'), t') \rangle \epsilon(t - t') \right\} \quad (8)$$

where the angular bracket denotes an average over the orientation \hat{n} .

The second term in the BP action leads to the thermal mass for the fermion. The BP appears to be chiral invariant under the transformation

$$\psi_\beta(\vec{x}, t) \rightarrow e^{i\beta\gamma_5} \psi_\beta(\vec{x}, t) \quad (9)$$

The Noether charge[9] associated with this chirality is however not given by eq.(5) but⁵

$$Q_5^\beta = \frac{1}{2} \int d^3x \left\{ \psi_\beta^\dagger \gamma_5 \psi_\beta - \frac{T'^2}{8} \int dt_1 dt_2 \epsilon(t_1 - t) \epsilon(t - t_2) \right. \\ \left. \psi_\beta^\dagger(\vec{r} + \frac{\hat{n}}{2}(t_1 - t_2), t_1) (1 + \gamma_o \vec{\gamma} \cdot \hat{n}) \gamma_5 \psi_\beta(\vec{r} - \frac{\hat{n}}{2}(t_1 - t_2), t_2) \right\} \quad (10)$$

To understand further the physics of this Noether charge, it is necessary to quantize the BP action.

The BP action is manifestly nonlocal. There is an essential difference between this nonlocality and the situation at $T = 0$. There the nonlocality is weak, since they are protected by appropriate powers of the cutoff Λ in the denominator. A derivative expansion thus makes sense if one is talking about physics at a momentum scale below the cutoff. For high T , however, the nonlocality is proportional to T^2 *in the numerator*, and no derivative expansion is possible.

³ Since R. Pisarski has already covered it in his lectures here at the workshop, I will conserve space and not describe the model here in any detail.

⁴ In real time formalism, there is a contribution to the full action from the $\tilde{\psi}$ associated with the heat bath. It is given by the tilde operation acting on the action in eq.(8), such that $I_{full} = I_{BP}\{\psi\} - I_{BP}\{\tilde{\psi}\}$

⁵ Likewise, the complete charge includes an identical but negative contribution where ψ everywhere has been replaced by $\tilde{\psi}$ field.

Because of the nonlocality, ψ_β is not a canonical field in the BP action.⁶ To quantize this action, it is convenient to work in momentum space, where the action takes the form

$$I_{\text{BP}} = \int d^4p \left\{ -i\bar{\psi}_\beta(p) (\vec{\gamma} \cdot \vec{p} - \gamma_o p_o) \psi_\beta(p) + i \frac{T'^2}{2} \bar{\psi}_\beta(p) (\vec{\gamma} \cdot \vec{p} a - \gamma_o p_o b) \psi_\beta(p) \right\} \quad (11)$$

with $a \equiv \frac{p_o}{2p^3} \left| \frac{p_o+p}{p_o-p} \right| - \frac{1}{p^2}$ and $b \equiv \frac{1}{2pp_o} \left| \frac{p_o+p}{p_o-p} \right|$

It may be checked that this action gives rise to the fermion propagator $\langle T(\psi_\beta(\vec{x}, t) \bar{\psi}_\beta(0)) \rangle_\beta$ (see footnote 6) with the usual analyticity properties, *viz.* positive and negative energy poles from *both* particles and holes of mass T' , plus a parallel pair of conjugate plasmino cuts in p_o plane that extend from $-p$ to p just above and below the real p_o axis. The discontinuity across each cut is of order T'^2 .

For our discussion here, we shall work to order T' and ignore the contributions due to the plasmino cut. The canonical field Ψ may be obtained by a redefinition of ψ

$$\psi(p) = e^{i \frac{T'}{2} \Theta} \Psi(p) \sqrt{z_p} \quad (12)$$

where $\Theta \equiv \vec{\gamma} \cdot \vec{p} a - \gamma_o p_o b$, and z_p is the wave function renormalization, which to order T' is simply unity. With this field redefinition, we find

$$I_{\text{BP}} = \int d^4p \left\{ -i \bar{\Psi}(p) (\vec{\gamma} \cdot \vec{p} - \gamma_o p_o) \Psi(p) - T' \bar{\Psi}(p) \Psi(p) \right\} \quad (13)$$

confirming that Ψ indeed is the canonical massive Dirac field for the BP action. To specify the $\pm i\epsilon$ boundary conditions, we require that at $t = 0$, Ψ field coincides with the free massless ψ_β field in a thermal equilibrium, so that equations (2,3) hold with mass gap equal to T' , and likewise for the tilde degrees of freedom. If we work with the Bogoliubov transformed basis

$$|0\rangle_\beta = \prod_{p,s} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} a_{p,s}^\dagger \tilde{a}_{-p,s}^\dagger \right) \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} b_{-p,s}^\dagger \tilde{b}_{-p,s}^\dagger \right) |0\rangle \quad (14)$$

the new vacuum is a generalized NJL vacuum

$$|vac\rangle_\beta = \prod_{p,s} \left(\cos \theta_p - s \sin \theta_p a_{\beta,p,s}^\dagger b_{\beta,-p,s}^\dagger \right) \left(\cos \theta_p - s \sin \theta_p \tilde{a}_{\beta,p,s}^\dagger \tilde{b}_{\beta,-p,s}^\dagger \right) |0\rangle_\beta \quad (15)$$

where $a_{\beta,p,s}^\dagger, b_{\beta,p,s}^\dagger$ are the Bogoliubov transform of the usual massless operators.

The full Noether charge Q_5^β may now be expressed in terms of the canonical annihilation and creation operators of the massive Dirac field as

$$Q_{5\text{ full}}^\beta = -\frac{1}{2} \sum_{p,s} s \left(A_{p,s}^\dagger A_{p,s} + B_{-p,s}^\dagger B_{-p,s} - \tilde{A}_{p,s}^\dagger \tilde{A}_{p,s} - \tilde{B}_{p,s}^\dagger \tilde{B}_{p,s} \right) \quad (16)$$

If you compare this with the canonical expansion for the $T = 0$ Noether charge, eq. (5), you'll see why Q_5 does not annihilate the generalized NJL vacuum, while the Q_5^β (expressed in terms of the massive quasiparticle operators) does.

⁶ Note that $\langle T(\psi_\beta(\vec{x}, t) \bar{\psi}_\beta(0)) \rangle_\beta$ is by definition also equal to the thermal average of the Heisenberg fields $\sum_n \langle n | T(\psi(\vec{x}, t) \bar{\psi}(0)) | n \rangle = e^{-\beta E_n} / Z$. Therefore it is reassuring to verify that the vacuum expectation $\langle \{\psi_\beta, \psi_\beta^\dagger\} \rangle = \delta(\vec{x} - \vec{y})$, even though ψ_β does not satisfy it as an operator identity.

5 Pion Halo

I conclude my talk by showing you how a QCD pion at high temperature can propagate as a massless particle and yet have a screening mass proportional to T . In the language of real time thermal field theory, it is easy to find an example of such a particle. For the physical massless pole is determined from the condition that the denominator of the propagator vanish

$$\Gamma_{\pi}^{(2)}(p, p_o, T) = p^2(1 + \mathcal{A})^2 - p_o^2(1 + \mathcal{B})^2 = 0 \quad (17)$$

where \mathcal{A} and \mathcal{B} are functions of p, p_o, T . The screening mass on the other hand comes from integrating over the x, y, t coordinates (i.e. set $p_x = p_y = p_o = 0$) in the propagator, so that the pole for the correlation function in z occurs at $p_z = im_{sc}$, where $1 + \mathcal{A}(im_{sc}, 0, T) = 0$. In terms of a physical picture, when we receive light from a charged particle, we see it at its retarded position, and it is a sharp image. For the pion, the retarded function reads

$$D_{\text{ret}}(\vec{x}, t) = \theta(-t) \left\{ \delta(t^2 - r^2) + \frac{T}{r} \theta(t^2 - r^2) \left[e^{-T|t-r|} + e^{-T|t+r|} \right] \right\} \quad (18)$$

so that the screening mass leads to an accompanying modulator signal that ‘hugs’ the light cone, with a screening length $\propto 1/T$.

References

- [1] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); *ibid.* **124**, 246 (1961).
- [2] A. Anselm, M. Ryskin *Phys. Lett.* **B 226**, 1991 (;) J.-P. Blaizot, A. Krzywicki *Phys. Rev.* **D 46**, 246 (1992); J. Bjorken, *Int. J. Mod. Phys* **A 7**, 4189 (1987), *Acta Physica Polonica* **B23**, 561 (1992); K. Kowalski, C. Taylor preprint hep-ph/9211282 (1992); J. Bjorken, K. Kowalski, C. Taylor SLAC preprint SLAC-PUB-6109 (1993).
- [3] J.C.Taylor and S.M.H.Wong, *Nucl. Phys.* **B 346**, 115 (1990); E. Braaten and R. Pisarski, *Phys. Rev.* **D45**, 1827 (1992); J. Frenkel and J.C. Taylor, *Nucl. Phys.* **B 374**, 156 (1992).
- [4] A. Gocksch and A. Soni, *Z. Phys.* **53**, 517 (1992).
- [5] L.N. Chang, N.P. Chang, *Phys. Rev.* **D45**, 2988 (1992).
- [6] H.A. Weldon, *Phys. Rev.* **D26**, 2789 (1982); V.V. Klimov, *Sov. J. Nucl. Phys.* **33**, 934 (1981).
- [7] J.F. Donoghue and B.R. Holstein, *Phys. Rev.* **D28**, 340 (1983); *ibid.* **29**, 3004 (E) (1984); L.N. Chang, N.P. Chang, K.C. Chou, *Phys. Rev.* **D43**, 596 (1991). See also G. Barton, *Ann. Phys.* **200**, 271 (1990). In contrast with ref. [6], the authors here take the perturbative approach and regard T' as a small parameter. The difference is negligible for $T \gg p \gg T'$ in the range where p is still ‘soft’.
- [8] L.N. Chang and N.P. Chang, *Phys. Rev. Lett.* **54**, 2407 (1985). See also H. Munczek, *Z. Phys.* **C32**, 585 (1986).
- [9] A.H. Weldon, Proceedings of Winnipeg Summer School 1992, Canadian J. Phys. See also J.P. Blaizot, E. Iancu, *Soft Collective Excitations in Hot Gauge Theories*, SACLAY-SPHT-93-064, Jun 93.